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EDITORIAL.

H.C.F. The methods spoken of in Mr. Hart's paper are excellent. The addition and subtraction method can be used even in Arithmetic, if the numbers to be factored are large. This method, however, allows still farther development.

If the H.C.F. of two cubics is to be found, eliminate the cubes, and factor the resulting quadratic; if of a cubic and a quartic, eliminate the fourth power in the quartic by use of the cubic times the letter of which it is a function, and the third power in the quartic by the cubic, and factor the resulting quadratic. Similarly any two functions can, by successive eliminations, of the highest powers, be reduced to a quadratic, unless the H.C.F. is of higher degree than the second, in which case it will show its presence by reducing the other function to zero.

This then, is a general method of finding the H.C.F. of two functions. It is, however, only the Euclidean method with two variations: it carries the method only as far as is necessary in order to get a factorable form, stopping, in general, at the quadratic; it uses elimination by subtraction, instead of division with its use of successive remainders. That elimination by subtraction is an easier idea than that of the division method can hardly be doubted; the proof also is simpler than in the other method. A comparison of the two methods gives an excellent idea of division as a form of subtraction.

Since the Euclidean method, with its ability to handle com-

plicated examples in H.C.F., has, by common consent, been dropped from the curriculum, it would seem that one should not be tempted to carry even this more easily explained method too far, although it is of undoubted value in working out questions that are still of accepted type. S.

A TASK.

To be honest, to be kind, to earn a little and to spend a little less; to make upon the whole a family happier for his presence; to renounce when that shall be necessary and not be embittered; to keep a few friends, but these without capitulation; above all, on the same given condition to keep friends with himself; here is a task for all that a man has of fortitude and delicacy.—*Robert Louis Stevenson.*

HIGHEST COMMON FACTOR.

By Howard F. Hart.

As the colleges now are requiring highest common factor by factoring methods only, any plan whereby the number of factors to be tried can be lessened is certainly worth while. For in general we must regard as possible any binomial factor whose first-degree term is an integral divisor of the highest term of the given expression and whose independent term is an integral divisor of the independent term of the expression. Thus in such a problem as, "Find the H.C.F. of $5x^3 - 21x^2 + 5x - 4$ and $5x^3 - 19x^2 - 5x + 4$" (McCurdy's Exercise Book, page 40, example 4) the possible factors that a student might try and must try, unless he were very lucky in those he chose to try first, are $x \pm 1$, $x \pm 2$, $x \pm 4$, $5x \pm 1$, $5x \pm 2$, $5x \pm 4$. And further if the given expressions were, say, cubics having no common binomial factor at all but with a quadratic one instead (*e. g.*, $2x^3 + 5x^2 + x - 3$ and $2x^3 - x^2 - 5x + 3$) I doubt if the ordinary first-year student would get any result unless it were unity.

As definite methods in eliminating factors and thus reducing the labors of factoring by trial I have found the following to be very useful as engines in attack. With one small exception (viz., a cubic having always at least one real root) even the proofs are not above what a first-year student can grasp.

First, if any one of the given expressions is a quadratic, that, or either of its factors, or unity are the only possibilities. And as quadratics are readily factored by inspection, if they can be factored at all, there is no real problem in highest common factor unless the given expressions are all at least cubics.

Sometimes one of the given expressions can be readily factored even though the others cannot. In such a case there is no real problem either as it is merely a question of trying the factors of the factored expression. And as we shall see in a moment we might not have to try all of those.

So our problem is: Given two (in case three or more expres-

sions are given we take them two at a time) cubics or higher forms neither of which can be readily factored to find the product of their common factors.

In the first place we should note that if a cubic can be factored at all there will be one binomial factor (corresponding to its necessarily one real root).

And in the second place it is obvious that in cases where the highest terms differ in coefficients or the lowest terms are different that certain apparently possible factors of each expression cannot be common factors and hence need not be tried. Thus in the problem "$x^5 - 2x^4 + x^2$ and $2x^4 - 4x^3 - 4x + 6$" (Sheffield) $x \pm 2$, $x \pm 3$, $x \pm 6$, $2x \pm 1$, $2x \pm 3$, $2x \pm 6$ cannot be common factors. Therefore all binomial factors but $x \pm 1$ or $(2x \pm 2)$ have been excluded without the labor of trial. Regarded as a point in division even the dullest students appreciate, it will be found, *why* such factors can be excluded at once.

These two methods between them often reduce the problem to the consideration of one binomial factor.

Still there are cases where, unless the student can in general factor at least one of the given forms completely he never knows but that there might be other common factors which he has not found.

In this connection the theorem, "Any *common* factor of two quantities is a factor of the sum and difference of the given quantities or multiples of them" is of great aid, for often the sum or difference is a vastly simpler function than either of the given ones and readily factorable. And its proof is so simple that first-year students can comprehend it—particularly if it is taken up as a generalization of many previous tested cases.

Let E_1 and E_2 represent the expressions, f the product of their common factors so that

	$aE_1 = afX$	where X and Y represent
and	$bE_2 = bfY$	the products of the re-
then	$aE_1 \pm bE_2 = f(aX \pm bY)$	maining factors.

The same special cases will make it clear that the converse is unfortunately not a true theorem.

To illustrate its use suppose we apply it first to the problems given above in the first paragraph. In the first one on adding we have $10x^3 - 40x^2$ or $10x^2(x-4)$ and as we know that the desired result is the product of the common factors only and that every factor common to the given expressions is a factor of $10x^3 - 40x^2$ we merely have to discover whether $x-4$ is a common factor or not.

In the second example addition of the forms gives $4x(x^2+x-1)$ and as x^2+x-1 is discovered by trial to be a common factor it is therefore by the same logic as above the H.C.F.

Of course wherever H.C.F. is needed this method will apply. Thus, suppose we are asked to reduce $\dfrac{x^3-5x^2-2x+24}{x^3-4x^2-3x+18}$ to lowest terms. If we subtract we obtain $-(x^2-x-6)$ or $(-1)(x-3)(x+2)$ and trying both $x-3$ and $x+2$ as divisors we get as a solution, $\dfrac{x-4}{x-3}$.

And again. If the relation $\dfrac{E_1E_2}{\text{H.C.F.}} = \text{L.C.M.}$ is used as the method of finding the L.C.M. that problem becomes merely a problem in H.C.F. and therefore as readily solved.

Finally we might note that if the given expressions have unlike coefficients it is sometimes possible to get a simple sum or difference by taking multiples as, *e. g.*, "To find the H.C.F. of $4a^5+14a^4+20a^3+70a^2$ and $6a^6+21a^5-12a^4-43a^3$" (Robbins and Somerville Exercise Book, page 42, example 11). If we remove the common factor a^2, multiply the first by $3a$, the second by 2, and subtract we have $42a(2a+7)$. On trial we find that $(2a+7)$ is a common factor so that the solution is $a^2(2a+7)$.

In all of the above cases the trial of the suggested factors is best made by synthetic division for an exposition of which read Mr. E. R. Smith's article in the MATHEMATICS TEACHER for September, 1910.

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TRAINING FOR EFFICIENCY IN ELEMENTARY MATHEMATICS.

By E. H. Koch, Jr.

To-day the dominant note in education is efficiency. It is fitting therefore that we should consider the factors for training in efficiency in elementary mathematics. The work of this association like that of kindred societies affords the best opportunity for promoting efficiency. In this respect our work does not differ from the main work of every other world movement as manifested in the conventions and meetings of men with their coworkers. In fact the agitations for improvement in the teaching of mathematics is as you know but one phase of a perplexity which confronts every field of human activity.

The expressions of opinions, serious comment, great thoughts, inventions, social and intellectual achievements are not spontaneous nor do they occur in a single mind because they are results of race development. How often in our experience have we been spared from expostulating on some cherished idea, design, method or invention because accident has given priority to some other independent worker. Such first announcements are apt to be vague and undefined and as a consequence men take issue and often give way to passionate disregard and intolerance for the tenacious attitude of an adversary.

I have a pretty firm belief in that greatest of paradoxes, viz., that both sides of an argument are right. Therefore, my friends, I am going to regard your assertions relating to the cultural and utilitarian value of any mathematics material as amphibolous aspects of an ideal future vocation. I am not going to reconcile these aspects, because they need no reconciliation. If you will permit me to state an analogy it appears to me as if some of us had gone into a long glass tube while the others remained outside. In so doing we have chosen our limitations of vision. When we recognize the fragile barrier that separates our groups we can understand why we look upon one another telescopically or microscopically. We owe to a heritage the

fallable belief in a dualism which has been symbolized constantly in the development of our literature, art, religion and science. The sense of opposites expressed as north and south, hot and cold, life and evil suggest culture and utility. Let us give praise to that noble soul who first annunciated the law of relativity. Now you know there is no greater avenue of adventure than higher geometry which reveals the infinite attenuation of life and comprehends the processes which guide the unborn possibilities of the future.

The isolation of the subjects of the old curricula has produced sterility.

"Education has not been able to keep abreast with the innumerable problems of our complex life very largely because its emperical methods are obsolete."

"Even scientific men are recognizing the fact that they must apply scientific methods to determine the circumstances which promote or hinder the advancement of science."* It is surely time for educators to apply the scientific method to determine those factors which enter into the problem of the efficiency of educational systems and more particularly to the special problem regarding the training for efficiency in elementary mathematics.

The best definition of efficiency is borrowed from the technical man who first ascribed it in a commercial way to a machine. Efficiency is the ratio of net output to gross input.

$$(a) \qquad \text{Efficiency} = \frac{\text{output}}{\text{input}}.$$

The human being is also a machine in the sense that it is a highly organized complexity of more or less coördinated parts. The intellectual attributes when associated with the nervous system bear to the framework of the body the analogous relation that the forces of nature bear to structural material.

Efficiency is a measure and is therefore expressed as a ratio between a realized performance and an ideal possibility. There are as many different aspects of efficiency as there are considerations in any mental or physical situation. When you speak of health or strength you unconsciously give efficiency a special name.

* *Science*, November 4, 1910.

It was quite natural to expect technical men to apply this term to the situation which presents itself in technical and industrial education, because the facts in their case were more definitely known. In this respect they have made a beginning.

It is my purpose to-day to indicate how such an application can be made in the special field of mathematics. Let us consider the summary of our technical colleagues who volunteer the following meanings of efficiency applied to education:—*

1. The Efficiency of the Curriculum—the ratio of the useful information and mental training offered by a college to that which an ideal college should offer

$$(1)\ E_C = \text{Efficiency of curriculum} = \frac{\text{what is offered}}{\text{what should be offered}}$$

2. The efficiency of the teacher—the ratio of the amount of knowledge and mental development acquired by the student to the amount of the opportunities offered by the college

$$(2)\ E_T = \text{Efficiency of teacher} = \frac{\text{what and how he teaches}}{\text{what and how he should teach}}.$$

3. Efficiency of the student as receiver—the ratio of that which the student assimilates to that which the college offers.

$$(3)\ E_R\ \text{Efficiency of student as receiver} = \frac{\text{what he assimilates}}{\text{what the college offers}}.$$

4. Efficiency of the student as giver—the ratio of the use made by the student of his college education during his professional career, to the potential energy stored in him at the time of graduation.

$$(4)\ E_G = \text{Efficiency of student as giver} = \frac{\text{what he gives to the country}}{\text{what he has received from the college}}.$$

There was a time when the soldier was the most efficient man

* Karapetoff, Contributions to the discussion of efficiency in engineering education, *S. P. E. E. Bulletin*, Vol. XVIII.

in the state. Centuries later the mantel of efficiency befel to the lot of the lawyer. The evolution of the political, social and industrial progress in the development of civilization compels the finger of time to point to the men of science as the dominant leaders of the nations.

"In America at least," says Professor Townsend,* "we have come to accept as a fundamental principle that the supreme test of an education is the efficiency of the training it gives the individual to meet the demands of organized society and at the same time enable him to contribute most directly or indirectly to the general progress of national life." Therfore:

(5) E_M = Man's overall efficiency =

$$\frac{\text{his productiveness} + \text{his contributions to society}}{\text{latent possibilities times education in its larger sense}}.$$

(6) $$E_M = \frac{P + C}{LS}.$$

P = productiveness.
C = contributions to society.
L = latent possibilities.
S = education in its larger sense.

Each one of these quantities is resolvable into innumerable factors many of which although not definitely known are nevertheless familiar bywords in the parlance of educational meetings. The unknown factors are equally important and must be determined and investigated. The task at first thought appears so stupendous as to seem almost insoluble, but if we apply the scientific spirit we need not despair. Some years ago the government sought to devise an automatic machine for predicting the tides for any locality, and for long stated intervals in advance. The question was carefully undertaken, although the obstacles seemed as insurmountable as the elements entering into weather prognostications. The working equation contained thirty-five variables. To-day that machine is a realization. Our problem narrows down when we confine our attention to the elements of mathematics which contribute to the efficiency of the individual.

* *Science*, November 4, 1910.

The individual may be likened to the incandescent lamp which has an efficiency of approximately one per cent. This means a waste of ninety-nine per cent. of the energy stored in the coal. Briefly told the final efficiency is the product of all the contributing efficiencies involved in the mechanical and electrical transformations and transmissions through boiler, engine, generator, wires and filaments of the lamp.

$$\text{(A)}\quad E_S = \text{Efficiency of the system} = \frac{\text{energy given out as light}}{\text{energy stored in the coal}}.$$

$$\text{(B)}\quad E_S = \frac{\text{energy stored in the coal} - \text{total losses of energy}}{\text{energy stored in the coal}}.$$

In like manner man's overall efficiency may be expressed in terms of losses.

$$(8)\qquad E_M = \frac{LS - \{O + I + F\}}{LS}.$$

substituting in (6) and (B).

O = lost opportunities,

I = impediments,

F = failures,

$$(9)\qquad \therefore P + C = LS - (O + I + F),$$

substituting in (8) (6).

Or we can represent man's overall efficiency as the product of all the efficiencies in the educational power station.

$$(10)\qquad E_M = E_C E_T E_R E_G. \quad \text{See (1) (2) (3) (4).}$$

We are especially interested in the mathematics applied to E_C. We shall define mathematics efficiency as the ratio of the factors which contribute to man's social, political and industrial abilities to the stimulus and power given by mathematics training.

$$(11)\quad E_m = \text{mathematics efficiency} = \frac{\text{factors which contribute to social, political, industrial abilities}}{\text{stimulus and power given by mathematics training}}.$$

(12) $$E_m = \frac{X}{M}.$$

(13) $$X = f(x_1 x_2 x_3 x_4 \dots x_n),$$

(14) $$M = \phi(m_1 m_2 m_3 \dots m_k),$$

(15) $$\therefore E_m = \frac{f(x_1 x_2 x_3 \dots x_n)}{\phi(m_1 m_2 m_3 \dots m_k)}.$$

$x_1 x_2 x_3 \dots x_n$ = the factors affected by mathematics which contribute to man's social, political and industrial abilities,
X = a function of $x_1 x_2 \cdots x_n$,
$m_1 m_2 m_3 \dots m_4$ = the potential contributions of algebra, geometry, analysis and mechanics,
M = a function of $m_1 m_2 \dots m_k$.

There are two ways of increasing E_m: either increase the numerator X or decrease the denominator M.

The known m's are usually summarized as:

m_1 = the notions of number, measure, form, element,
m_2 = the focusing of law through symbols and graphics,
m_3 = the visualization and interpretation of law through formulas and graphic representation,
m_4 = the appreciation of coexistences and sequences of phenomena,
m_5 = the projection of the intellect beyond experience,
m_k = not defined.

We shall next turn our attention to the x's and challenge their validity. We have been talking about these x's for a long time and I dare say have invented a few out of mere defense for an educational hobby. We must not forget one of the staunchest and earliest critics, Professor Perry,* "who holds that the study of mathematics began because it was useful, it continues because it is useful and it is valuable to the world because of the usefulness of its results in

x_1 = producing the higher emotions and giving mental pleasure,
x_2 = brain development,

* British Association Meeting, Glasgow, 1901.

x_3 = producing logical ways of thinking,
x_4 = aid given by mathematical weapons in the study of physical science,
x_5 = passing examinations,
x_6 = giving men mental tools as easy to use as their legs or arms, enabling them to go on with their education,
x_7 = teaching a man the importance of thinking things out for himself and so delivering him from the present dreadful yoke of authority,
x_8 = making men ... feel that they know the principles,
x_9 = giving acute philosophical minds a logical counsel of perfection altogether charming and satisfying."

"Any subject may be useful as applicable to some special purpose or need of life or it may be useful as affording valuable mental discipline."

The following x's were excerpted from various sources:

x_{10} = mathematics should promote culture, which means the subjugation of imitative power to the creative so as to make the individual develop centrifugal force, individuality, critical opinion and transform that which is read into conversation and life.*
x_{11} = mathematics should maintain the standards of a liberal education which implies a sense of truth and beauty.
x_{12} = mathematics should promote the identical spirit of science and literature which is to transform something of value from the unknown into the realm of the known.
x_{13} = mathematics should respond to the dominant activities of the nation.
x_{14} = mathematics must justify itself in the acquisition of information of commercial value.
x_{15} = mathematics should provoke intellectual tolerance—give breadth of thought.
x_{16} = mathematics should stimulate enthusiasm.
x_{17} = mathematics should instill courage and sympathy with life.
x_{18} = mathematics should give a realization of the value of human life as a public asset.

* Address Professor Aston, Columbia University, September 28, 1910.

x_{19} = mathematics should promote intelligent leadership in matters of public concern.

x_{20} = mathematics should promote economy of time and effort.

x_{21} = mathematics should promote diligence, conscientiousness.

x_{22} = mathematics should promote strength of character.

x_{23} = mathematics should promote judgment.

x_{24} = mathematics should promote fidelity, poise.

x_{25} = mathematics should promote neatness and accuracy.

x_{26} = mathematics should train in habits of attention.

x_{27} = mathematics should train in system in dealing with material.

x_{28} = mathematics should train in clear and interesting presentation both oral and written.

x_{29} = mathematics should train for foremanship, management and other executive positions.

x_{29} = mathematics should render service in the development of the national resources in aiding the growth and expansion of its industries and of its commercial power and in the conservation of the resources that constitute the inherited wealth of a people.*

x_{30} = mathematics should furnish a solid foundation on which the more advanced work of college and technical school may be based.

x_{31} = mathematics should develop an appreciation of the methods and spirit of pure science.

x_{32} = mathematics should lead to precision.

x_n = last but not least mathematics should promote common sense.

There are many more factors of the numerator which may be mentioned. The critical test must be applied to each x, the so-called desirable accomplishment, to see if it rings true or whether it has been coined to defend in a vague way some pet fancy of the mathematics department. Is it not generally true that in determining the efficiency of any branch of study these same accomplishments are more or less in advocacy. It seems to me

* See Science and Public Service Address University of Illinois Biologic Station, July 22, 1910. "American Educational Defects," *Science*, October 28, 1910. Report of Training of Mathematical Instructors, *Bulletin A. Math. Soc.*, November, 1910.

that if this were true we would be wasting effort and time in accomplishing in an inefficient way what may be more efficiently accomplished in another subject.

Beyond the ordinary claims mentioned which have been advanced and reiterated is the potent claim of a very broad consideration. I am convinced that in every department of human investigation there are definite underlying mathematical relations which are either little understood or totally unknown.

It was a great pleasure for me some two years ago to listen to a score of weekly addresses delivered by one of the representatives of each of twenty-four departments of study at Columbia University. In the period of one hour at the disposal of each of these scholars he gave a brief summary of the historical development of his subject with the emphasis on its important contributions to mankind and a hint of future anticipations. I do not believe I have listened to any other series of lectures with a like interest, not because I fully understood their comprehensiveness, but because of the acute sense of the mathematical basis and generalizations underlying these subjects. If you will permit me to be a little more personal I would state that it is a phase of investigation that I have been interested in for a number of years and I owe my inspiration to our esteemed colleague Dr. Schwatt.* It was extremely encouraging to have so many of these representative men acknowledge at least a meagre indebtedness to the mathematical elements in their subject. Someone has said "a man's capacity for mathematics is paralleled with the development of the mathematical activity present in the great departments of investigation." Formerly the subject, biology, was considered remotely distinct and separated from mathematical analysis. Then those students who were not inclined to the mathematical discipline of their courses flocked to biologic studies only to learn that they had chosen intensely mathematical subjects. It is for this reason that most of our biologic activities are scarcely more than the clerical work of classification and cataloging.

It seems to me that our next great work as mathematics teachers will be to lend our influence to the discovery of the relationships which underlie other fields of research. If these

* "Some Considerations Showing the Importance of Mathematical Study," Philadelphia, 1895.

relationships belong in any mathematical category as I believe they do then we have a large problem whose solution shall not only conserve the mathematical field but shall add the greatest stimulus to mathematics training.

Let us determine in how far each of the arbitrary divisions of the subject matter of mathematics contributes to desirable human achievements. Let us give proportionate weights to these divisions which we designate as algebra, geometry, analysis and mechanics and determine whether the time spent upon them is justified or whether the raw product, the material of our text-books, is poor or whether we are extravagant or indulgent in the luxuries of our subject. It is generally conceded that mathematics knows no poverty. If therefore we should discover that our arbitrary division of mathematics is wrong let us have the good sense to change it, and if we have the conviction that the subject itself is wrong let us have the courage to throw it out. We must recognize the cold hard fact that we are dealing with a commercial problem in which we shall be harder pressed. If we can give no adequate answer then we are no better than the unskilled laborer in the street. The tendency in education is democratic not aristocratic, as men have vainly attempted to make it in the past. Our ranks are swollen with men and women whose ideals and culture are no higher than those of despised workers in other vocations. Professional life is no longer a guarantee of the homage of exaltation. Why is it? Because we have introduced so much in our studies that is unreal. We have deceived ourselves into believing that our efficiency was measured by the number of students graduated or passed to a higher grade. What about all those young men and women who have been neglected because we thought their withdrawal from school or college was an expression of unfitness. Yes, an unfitness in most cases for an inefficient school system. Now as mathematics teachers how heavily have we played in this rôle. We must not be amazed then if we hear a challenge put to our cherished subjects. Will algebra, geometry, trigonometry, analytics, calculus, even arithmetic, stand the test when we ask without bias are they taught efficiently in their present text-book form. They have served us well in the past but had we not better tear them

down .nd build new structures in their places? If we are content t.) leave them remain for a longer time we must repair and replenish them. Most of our teachers look upon algebra, geometry and trigonometry as if they were the body of materials embodied in a standard text-book on that subject. These are the halt but we have just as many blind teachers who regard the formula as wooden as if it were intended merely for substitution from data instead of for its larger purpose of interpretation. These impediments to efficiency are the result of our failure to recognize the world as a domain of relationships. It does seem timely that we should incorporate these subjects and teach them more efficiently under one organized whole. I believe we can teach mathematics more efficiently by presenting theory through the problem and postponing it until we need it and can make immediate use of it. In this way we can avoid a great amount of the drudgery of algebra. I see some of you staggering under the hints of corporate aggrandizement. Nothing of the kind, my friends. It is simply the motherlove of mathematics applied to the correction of the errors of her brood.

"As each subject grows it fringes" and in the garden of mathematics there are weeds to pull. One of the most persistent weeds that grows among the flowers in our estate is the so-called "problem." Why do we permit these monstrosities, these unreal problems to fill our text-books? They represent a type of fiction which is revolting to the child. They are vicious in principle because they contradict the realities of life. They negate that highest virtue of mathematics, the search for truth. We defined efficiency as a ratio, a fraction. Every puzzle, or "faked" or "unreal" problem inserted in our text-books increases the denominator of our fraction and therefore decreases the efficiency. We should establish the principle never to insert a problem which has to be worked backwards in contradistinction with the realities of life. There is plenty of good material for real live problems but if our knowledge of real things is too limited we had better omit the fictitious problems entirely. I am reminded of a problem in a recent text-book which must have been written during a house cleaning epidemic. "My window is a yard wide. From a brass

curtain rod I cut off one sixth of its length and finding it still too long cut off one fifth of the length remaining. But I had to cut it the third time, taking off one tenth of its length to make it fit. What was the length of the rod?" Now no sane person in the world would ever be concerned about such a shade proposition. I tell you, ladies and gentlemen, our mathematics must be diseased when we have to insert such stuff. To make ourselves at home let us refer to the problems given by the College Entrance Examination Board in June, 1910. Having been a mathematics reader for the same board for eight years I am loath to criticize excepting that it be interpreted in the light of helpfulness, justified by experience. Mathematics Paper AI contained two problems. (1) "An estate of $36,500 is divided among 5 men, 4 women, and 6 children. A man and a child together get as much as 2 women. Four children together get as much as a man and a woman together. Find how much each man, woman, and child, respectively, receives; if all men are treated alike, all women alike, and all children alike." Think of that problem given in a democratic country. This is an obsolete and vicious problem. It is not constructive but destructive in so far as it negates common law and truth. It had better be omitted and thereby give the paper a better moral tone. We haven't time for puzzles in a crowded curriculum. The second problem reads: "A motor car traveled 20 miles at a certain uniform rate and then returned over the same route at a different uniform rate, the running time for the round trip being 2 hrs. 15 minutes. One third of the outward trip and one half of the return trip together occupied 55 minutes. Find the two rates of traveling." This is not a "real" problem but is a redress in modern equipment of an ancient text-book puzzle. Nobody would ever meet a "problem" of this kind and therefore it is a "type" that causes our students to lose respect for their mathematics. The biggest fool on earth wouldn't run "a motor car at a certain uniform rate and then return at a certain different uniform rate!" and the fractions of each trip would never disturb the calculating minds of the greatest or least mathematicians who rode in such a car.

Ladies and gentlemen, I maintain that it is a disgrace in this time and age to continue to suffer under this yoke of authority

and I demand in the name of the thousands of students who are preparing for college and especially the technical school that you give a voice of protest against this infliction.

There has been reason for honest variance of opinion regarding the cultural and utilitarian value of mathematics as long as these claims remained entrenched in the old undisputed doctrines. "There is no subject labelled cultural which is not at the same time practical and there is no subject labelled practical which is not at the same time cultural." Through mutual understanding we realize that education is beginning to teach subject matter in terms of daily life and is recognizing the factors which are of value to our people. We have made the first serious attempt in the history of American education to meet the demands of our economic, industrial and social problems. We have learned that tremendous industrial forces have been developed without any commensurate adjustment of our school system. We may no longer disregard the significance of industry in its influence upon our life. Our systems and subject matter must be rehabilitated. Out of the prevailing systems there have emerged forms of practical mathematics which connect the school with the industries and vocations of our people. They involve a more or less special training which is best adapted to make the student discover himself and to promote the qualities essential to his success in a specific vocation. We are getting a little closer to the various types of mind and the precise factors of intelligence.

Two years ago the speaker undertook the unification of the essentials of elementary mathematics for electrical students. The subject matter of this practical mathematical extends over the usual range of elementary mathematics including the notions and symbolism of the elements of calculus and vector analysis and vector applications. This was necessary in order to meet the students' requirements in both direct and alternating current work. All non-essentials are omitted and the teaching of theory is postponed until the student requires it in his work. We count this a great gain in economy. The student has a clear understanding of his work. He can express ideas intelligently. He has the power of applying knowledge practically. He realizes the right thing to do in an emergency. He

accomplishes a reasonable amount of work in a definite time. His class room is always a work room. His work is interesting. It gives him pleasure. He has greater physical endurance, poise, displays greater fidelity, assurance, neatness, accuracy and the greatest of all blessings, common sense. He is also trained for leadership and executive positions by imposing upon him responsibility of foremanship work in the class room, laboratory and shop. Upon entering the class room he immediately reports his outsigned work on a time sheet on the front cover of his work-book and makes a similar entry at the close of the period. The work-book contains every scrap of work which the student has completed and this book is retained by the instructor who has the students' entire work under constant supervision and inspection. The work of the instructor is made more efficient by the coöperation of the students through their foreman. We are simply recognizing a well-established principle that a man can learn to direct others by first learning to be directed by others. Secondly he can learn to guide others best by working with them, sharing their trials and disappointments and gaining an intimate appreciation of and sympathy with their modes of thought and action. "Of the many evils which our examination system has inflicted upon us the central one has consisted in forcing our school and university teaching into moulds determined not by the true interests of education but the mechanical exigencies of the examination syllabus."*

The time is not far distant when our college and state examination boards will recognize these facts and they will be compelled to offer examinations in a unified course such as practical mathematics. These demands will come first from the technical schools.

We cannot all be great mathematicians, and so our universities would become more efficient if they did not attempt to duplicate work which could be undertaken better by a few institutions. I wish we would hear the phrase institutions for teaching even more often than we hear about institutions for learning. Would it not be the part of wisdom if many of our institutions devoted their time in discovering how mathematics is contributing to other fields of intellectual activity and thereby

* *Science*, September 23, 1910.

promote such projects. We have seen an increased efficiency in mathematics teaching since history stepped in and opened a new pathway. We should establish experiment stations not only in agriculture and engineering, but also in mathematics and language and other subjects as well. Then the study of higher mathematics will have an immediate justification as well as blazing the future pathway of knowledge.

PRATT INSTITUTE,
BROOKLYN, N. Y.

COMPANIONSHIP.

But surely it is no very extravagant opinion that it is better to give than to receive, to serve than to use our companions; and above all, where there is no question of service upon either side, that it is good to enjoy their company like a natural man.—*Robert Louis Stevenson.*

AN ARITHMETICAL TEST IN THREE PHILADELPHIA HIGH SCHOOLS.

By Jonathan T. Rorer.

Introductory.

It has often been stated that the modern high school pupil does not possess ordinary skill in the common arithmetical processes. He has met unfavorable criticism in this respect not only from the college professor, who reports him unable to perform correctly the easy numerical work of the laboratory, but also from the man of business who frequently claims that the high school boy who finds his way into commercial life, cannot even add, subtract, multiply and divide.

As an experiment to test the truth of the latter criticism, the following questions, without previous warning, were simultaneously put to twelve hundred students in three of the large Philadelphia high schools, to one hundred students in each of the four years of the course in each of the three schools.

The Question Paper.

Philadelphia, Nov. 17, 1910.

Read these Directions Carefully.

1. Solve each problem in space provided, directly underneath.

Note: The question sheet was 9×14 inches, and the questions were properly spaced.

2. Place all work on this sheet and indicate result clearly.
3. Write name and grade at top of reverse side of sheet.

1. Add the following:

34256
78242
52167
785862
1946
71234
8761
326504
361
651302

2. A man who has saved $142.96 still has $72.01 less than his brother has; how much have they together?

3. A has 35¾ yards of cloth; B has 5½ yards more than A; and C has 12¼ yards less than B. How many yards have they all?

4. How much do I gain if I buy 300 oranges at the rate of $1.25 per hundred, and sell at 25 cents a dozen?

5. From a farm of 800 acres I sell 10 per cent. to Mr. Johnson, and 20 per cent. of the remainder to Mr. Fleming. How much do I still own?

6. Multiply .052 by 108.04. Divide the product by 200. (Result to three decimal places.)

7. Find the interest of $800 for 9 yrs. 5 mos. at 6 per cent.

It is evident that the above questions are in no sense a test of high school work, nor even of the advanced, or more difficult arithmetic of the elementary grades, but it is believed that they cover fairly well the more common computations of business. If the accusation of the business man is correct that our school product cannot perform the fundamental operations with proper facility, this test should surely show it. It was not claimed that it would show much more. The results, however, do seem to show some additional points of interest. While the test was timed to one half hour, it was found that less than five per cent. of the pupils used more than 20 miutes.

Results of the Test.

The questions were each given the same credit in the marking, and each was marked either right or wrong, no partial credits being given under any circumstances. Each paper was then given an average in the usual way. The average of the individual averages was then obtained for each of the three schools, and for each of the classes (first, second, third and fourth year) in each of the schools. The percentage of correctness for each of the seven questions was also computed. These facts are shown below:

Average of the schools: School A, 85.6 per cent.
School B, 83.7 per cent.
School C, 74.5 per cent.

Averages by Classes in the Schools.

School.	First Year.	Second Year.	Third Year.	Fourth Year.
A (boys)	83.1	74.7	86.4	95.1
B (girls)	73.8	82.6	91.2	89.4
C (girls)	68.0	74.1	80.0	79.0

Percentage of Correctness in Answers.

Number of question	1	2	3	4	5	6	7
Percentage correct	90	91	86.3	89.5	86.9	69	87.3

Discussion of the Results.

The above figures show that the students tested were, at any rate, not disgraced. Some would maintain that the questions were so easy that no credit is due high school pupils for obtaining the above grades. If the business man's criticism is not answered, the above testimony is at least favorable to the schools.

The influence on these results of the course of study in mathematics in the several schools is of interest.

In school A, the boys tested were almost exclusively in the classical and scientific courses, their mathematics being algebra, geometry, algebra continued, and trigonometry. But two commercial course sections were tested, sixty students. These were fourth-year boys who had been drilled in commercial arithmetic during their second year, but whose training in algebra and geometry was less than half that of the classical and scientific students. It is noteworthy that these commercial students averaged slightly less than the classical and scientific students in the same school year.

In school B, the first-year girls study algebra, while the second-, third- and fourth-year girls, being in the commercial course, study arithmetic.

In school C, the girls tested study algebra and geometry during the first three years and arithmetic in the fourth year.

The minimum ability of the boys for such work during the second year was not unexpected. Many teachers have noted this to be a time in the boy's high school career when accuracy is attained with great difficulty. After this period, his improvement is continuous, and the maximum of 95.1 per cent. in the fourth year, may be considered quite satisfactory.

An unexpected maximum of the girls' ability would seem to be in the third year. In school B, the girls of the fourth year who had received two hours per week instruction in arithmetic for one year more did not do so well; while in school C, it would seem that the girls are more accurate in such elementary calculation before receiving high school instruction in arithmetic than they are after receiving it. However, the decrement of the fourth year in the girls' schools is so slight that it

may be assigned to accidental causes and it would therefore be unwise to draw any definite conclusion from it.

The important lesson to be learned from the percentages of correctness is that teachers should give more emphasis to decimals. The percentage of error in question 6, was more than twice that of any other question. The average of correctness, considering all questions, was 84.3 per cent. It will be noted that on all questions save the sixth, the percentage of correctness is fairly uniform. If the sixth question be omitted, the average of correctness would be 88.5 per cent.

Conclusions.

Considering the character of this test, it is believed we can draw these conclusions from the above figures when studied with reference to the mathematical training of the pupils involved.

1. The high school boys or girls do not forget the fundamental arithmetical operations and the criticism that they cannot do simple commercial calculations with reasonable accuracy and speed is unwarranted.

2. The study of high school mathematics, not exclusively arithmetic—algebra, geometry, trigonometry, does not interfere with facility in simple commercial calculation but, on the other hand, increases the ability of the students in accuracy and speed.

3. The formal study of arithmetic in the high school is not necessary that pupils may retain the elementary school training in the simple business calculations.

My thanks are due to Mr. Albert H. Raub, District Superintendent of Schools, who prepared the questions, and arranged the test in the three schools, and who has kindly placed the above at my disposal for discussion.

William Penn High School,
December 15, 1910.

THE NEW SYLLABUS IN ARITHMETIC FOR NEW YORK STATE.

By Charles A. Shaver.

Your president has assigned me the topic "The New Syllabus in Arithmetic for New York State."

In the preparation of a syllabus of this character, it is essential to have, first of all a clear idea of the ends to be attained through the teaching of the subject. The ends or aims in arithmetical study as I understand them are: (1) Performing operations, (2) solution of problems, (3) explanation of processes.

The first of these aims, viz., performing operations, is, in my opinion, the one that should be emphasized in the primary grades, in fact, I hold that it is the only one which the primary teacher should consider.

During the past few years we have had many theories and methods advanced for the teaching of elementary number, many of which have been imperfectly understood and applied and the results have been anything but satisfactory. The Grube method, the ratio method and many others have for a time held a prominent place in primary instruction. We have passed through or nearly through the development stage of number work in which we have entertained the fallacy that no facts should be acquired by children unless the principles and ideas underlying them are first made clear.

Some years ago one of the leading mathematical teachers in the country said that children who spent the first four to six years of their school life in the country succeeded far better in the elementary mathematics than their city-school trained cousins; that as a rule, the country boys and girls who enter our high schools are easily the leaders in algebra and geometry. He attributed these conditions to the fact that country schools are not troubled with much apparatus and that many of the teachers do not keep pace with the latest fads in number; that in these schools interest must be found in the thing itself and that teacher and pupil work together and find it.

That renowned teacher and educational philosopher, Dr. Emerson E. White, is on record as saying, "Were I to be responsible for a child's arithmetical attainments at fourteen, I should insist that his training in number the first three years of school be made as natural and simple as possible and kept largely free from attempted insights into abstract relations and premature efforts at analytical and logical reasoning and I should strongly hope that he might be permitted to reach the third school year unhampered by such logical terminology as *because*, *whence*, *hence* and *therefore*. If my pupil at the end of the third school year could add, subtract, multiply and divide simple numbers, I would confidently guarantee his future progress and attainments in arithmetic." And he further says, "Were I to be personally his teacher in grammar grades, I should be delighted to find a few processes, principles and applications out of which the juice had not been sucked in the lower grades."

There is cogent reasoning and sound philosophy in these statements and I think that no apology may be made for a course of study in arithmetic that is in keeping with the ideas therein expressed.

The first of the arithmetical aims mentioned—the performing of operations—is made the aim of the first three years' work in the course of study. To secure, within that time, accuracy and facility in addition, subtraction, multiplication and division, the following principles are advocated:

1. The memorizing of the facts of number.

2. Drill in the use of these facts until facility in operation is secured.

3. The elimination of all use of objects and devices that tend to convey the idea that addition is counting.

4. The elimination of all explanation, *i. e.*, of the reason why—in these fundamental processes.

5. The elimination of fractions and all other applications of number including the solution of problems, that the entire time of the teacher and pupil may be given to securing the aim in view, viz., accuracy and facility in the fundamental operations.

Touching the first of these principles—memorizing the facts

of number, it is well known that with the nine significant figures, we may form forty-five combinations in addition, subtraction and multiplication. The syllabus holds that these are to be committed to memory and that sufficient drill and practice be exacted to secure accuracy and facility in their use. The use of objects in this work beyond their simple use in consecutive counting is eschewed, as their use, especially the counting of two groups of objects into a sum instils the idea that addition is counting, which is wrong in principle and fatal in practice, fixing, as it invariably does the pernicious habit of counting the fingers in computation.

The recommendation that the Austrian method of subtraction be used, will, if followed, practically do away with the process of subtraction and greatly reduce the time of teaching the two operations of addition and subtraction.

Referring to the fourth principle enumerated in the foregoing I have to say that it it is the one of all others, which I wish heartily to indorse. Who can compute the wasted time and effort resulting from the futile attempt to teach a child why he carries in addition. This is the time to teach the art of computation, not the science of numbers. I hold that it is impossible to teach a child in the primary grades the science of the Arabic notation, but even if it could be done, it would be no argument for teaching it. What the child *ought* to do, not what he *can* do should here be the guiding principle. As Chancellor Payne says, "To say that we should memorize only what we understand is very much like saying that we should commit nothing to the stomach until it has been digested. We eat to the end that we may digest and we must confide material to the retentive power of the mind in order that the intelligence may find something to work upon."

The second aim in arithmetical study—the solution of problems—is first taken up in the syllabus in the fourth grade. The facts of number and facility in operations acquired in the first three years are now to be applied to the concrete arithmetical problems of life. The syllabus recognizes this as the point where abstract work ends and concrete work begins. To quote from the syllabus for the fourth year, "Simple fractions and equivalents. *Develop objectively.* Simple problems oral and

written connected with daily life. Original problem work by the children. Teach cancellation. Have all problems stated before being worked. Use cancellation whenever possible in the solution of these statements."

The adoption of the suggestion to state all problems before working will result in a great saving of time to both teacher and pupil. This compels a pupil to do his thinking first and his work afterwards. It enables a teacher to quadruple the amount of drill in a given time and saves much time in the inspection of work. It also makes cancellation which has heretofore been useless the chief agency in shortening computations.

It follows as a matter of course, that the third aim in arithmetical work—the explanation of processes should, so far as the mental capacity of the pupil to understand, allows, go hand in hand with the solution of problems.

I consider, as worthy of much praise, the emphasis which the syllabus places upon the thorough teaching of the aliquot parts of 100 and their use in decimals and percentage, also the suggested use of the Austrian method in the division of decimals. These suggestions, if adopted, will save much time and contribute to the great twin factors of efficiency in arithmetic—accuracy and facility.

The completion of the ordinary course of arithmetic at the end of the seventh year as required by the syllabus, demands the omission of non-essentials. It may be difficult to secure anything like unanimity of opinion regarding what is and what is not essential in arithmetic. However, in my judgment, the syllabus is to be commended for the omission of the greatest common divisor by the division method, the obsolete tables and work in denominate numbers, its suggestion to emphasize drill and practice with fractions of reasonable magnitude and its advococy of thorough oral and mental training. The omission of the Metric system is likewise, in my judgment, an elimination that has been too long delayed. Why should the grammar school teacher be obliged to teach what nine-tenths of her pupils will never use, and what the few who will have occasion to use, when the high school is reached, can there learn in a few lessons. And learn it there they must, for if studied in the grades, it will, from neglect of use, be forgotten long before the high school is reached.

In concluding I would make special mention of the suggestion made in the last half of the sixth year that the simple equation and the use of the unknown quantity x can be profitably used in the solution of some problems.

Many teachers and many text-book writers seem to think that the equation belongs to algebra, and must be carefully excluded from arithmetic. It is hard to determine how this thought ever gained currency. The elementary treatment of the equation may well begin where recommended in the syllabus and continued through the seventh grade. It is encouraging to know that many of our best arithmetics are preparing for this work. The use of the equation widens the range of problems and simplifies the solution of many. It adds to the power of the pupil and hence to his interest in the subject. It gives a facility in logical reasoning that can not be obtained without its use. It gives to the pupil who never enters the high school the power that comes from the concrete side of algebra. It furnishes the pupil with a convenient tool by means of which he can use and develop his reasoning power. The pupil has the right to an acquaintance with this powerful aid.

In the last analysis, any course of study must be judged by its results. However, if the ends to be attained are based on sound educational philosophy and the means set forth to reach those ends are such as produce power and skill in application, as I believe to be the case with the present course of study in arithmetic, the results can be clearly discerned from the outset.

REPORT OF THE MINNEAPOLIS MEETING OF THE AMERICAN FEDERATION OF TEACHERS OF THE MATHEMATICAL AND THE NATURAL SCIENCES.

The annual meeting of the Council of the American Federation was held at the University of Minnesota, Minneapolis, Minn., on Wednesday morning, December 28, at 10 A.M.

The roll call of the meeting showed seven delegates present, six from the Central Association of Science and Mathematics Teachers and one from the Association of Mathematical Teachers in New England. Since there was not a quorum present, it was announced that all business transacted would have to be ratified by written vote of the Council.

In the absence of the secretary, Professor F. L. Charles, of the University of Illinois, was elected secretary pro tem.

The minutes of the last meeting (Boston, 1909) having been printed, it was voted that their reading be dispensed with and that they be approved as printed in *School Science and Mathematics* for April, 1910 (Vol. X., p. 343).

The report of the secretary-treasurer was presented in print in Bulletin III., and it was voted that the report be approved as printed.

Reports were presented from the Executive Committee, the Committee on College Entrance Requirements in Mathematics and Science, the Committee on Logarithmic Tables, the Committee on Bibliography, the Committee on a Syllabus in Geometry, the Committee on a Mathematical Journal, and the Committee on the Unit in Chemistry. It was voted that these reports be received, printed, and distributed to all members of the federated associations, in order that the council and the associations might have opportunity to consider them before taking action on them. These reports are appended.

Reports were presented from six of the local associations. It was voted that these reports be printed and distributed. Reports are appended.

The nominating committee reported nominations for the offi-

cers of the federation for the coming year as follows: President, C. R. Mann, the University of Chicago; secretary-treasurer, E. R. Smith, Polytechnic Preparatory School, Brooklyn, N. Y.; members of the Executive Committee—Ira M. DeLong, University of Colorado; I. N. Mitchell, State Normal School, Milwaukee; Wilhelm Segerblom, Phillips Exeter Academy, Exeter.

It was unanimously voted that the secretary cast the ballot for the nominees. The secretary cast the ballot, and those nominated were declared elected officers for the coming year.

On Thursday morning, December 29, the federation held a joint session, with Section L, A. A. A. S. This meeting was well attended. The topic, "Testing Results of Teaching Science," was presented by Professor E. L. Thorndike; and Professor O. W. Caldwell described some experiments in testing that were in progress at the University High School in Chicago. Professor Thorndike's paper is herewith printed in full, and Professor Caldwell's will be printed as soon as the experiment is completed and the results ready for publication.

F. L. Charles,
Secretary pro tem.

Since the meeting at Minneapolis, the actions there taken have been submitted to the council by letter and ratified by written vote of the majority of the members.

E. R. Smith,
Secretary.

Report of the Executive Committee.

In presenting its report this year, your executive committee wishes first to review briefly the activities of the federation. The first preliminary meeting of delegates for the purpose of organizing this body was held in December, 1906. No organization was completed that year, but a committee was appointed to frame articles of federation and report in 1907. The articles of federation were presented to the delegates from the local associations at a meeting in Chicago in January, 1908. Before the organization could be completed, however, these articles had to be approved by each of the local associations. The year

1908 was spent in making the purpose of the federation clear to the local associations and in securing the approval of seven of them to the articles of federation. These seven associations paid dues that year to the amount of about $70, of which $50 was spent in paying for the printing and postage used in the work of organization up to November, 1908.

In November, 1908, the executive committee issued an eight-page bulletin, which was sent not only to all the members of all the associations that had joined the federation, but also to the members of those associations that were considering joining. This bulletin contained a brief statement of the proposed purpose and policy of the federation, a brief history of its organization, the announcement of the appointment of a committee to prepare a bibliography of science teaching, and the call for the annual meeting in Baltimore in December, 1908. The cost of printing and distributing this bulletin was about $65.

Thus the work of making known what the federation was for, and of interesting associations in it, occupied two full years, and cost $115. The work of advancing science teaching, for which the federation was formed, could not begin until this preliminary work was completed.

During the year 1909 six more associations joined the federation. A second bulletin, of twenty-four pages, containing the report of the meeting in Baltimore, the papers read there, together with the announcement of the appointment of committees on publication, on a syllabus in geometry, and on the relations between the high school and the college was issued that year. In the appointment of these committees the real work of the federation began.

This second bulletin was again distributed to all members of all associations which had joined the organization, and was given wide publicity in the educational press. The total cost of printing and postage was about $75. Your committee considered it necessary to print and distribute these bulletins in this way, because there is as yet no established means of publication which reaches all the members of all the associations. The establishment of such a means of communication is still one of the problems before the federation. Two committees are at work on it—the one on publications and the one on a mathematics journal.

A third bulletin of four pages has just been issued, containing the call to the meeting at Minneapolis, the announcement of the appointment of the committees authorized at the Boston meeting, the financial statement, and the list of associations which are now members of the federation.

During the past two years, 1908–09, dues were paid by the associations at the rate of five cents per member. In return for this each member received Bulletins 1 and 2, beside assisting in paying the expenses necessary to the organization of so cosmopolitan a body. Last year the associations voted to raise the dues to ten cents per member, as it was clear that the work of the committees that had been appointed could not be supported and their reports printed without the additional income which resulted from the increase in dues. Some of this money has been used in paying for printing and postage for the committees, but the greater part has been reserved for use in securing copies of the report of the committee on bibliography. This report has been completed and accepted by the U. S. Bureau of Education for issue as one of its special bulletins on education. Arrangements have been made to secure enough reprints of this report to send a copy to every member of each of the local associations in the federation. Requests for copies of this report have been received from England, Germany, India, the Philippines and Honolulu.

The committee on the relations between colleges and high schools is presenting an important report at this meeting. This report is the result of eighteen months of hard work on the part of this committee. It contains valuable suggestions as to the reorganization of those relations so as to make the conditions for science work in the schools more advantageous, and calls for a discussion of these suggestions by the local associations. This report should be printed and distributed as soon and as widely as possible, and the committee should be continued in order to follow up their suggestions and try to get them put into operation in as far as they are approved. As a result of his activities as chairman of this committee, Mr. Butler has been made a member of a committee of the Department of Secondary Education of the National Education Association, which has just been appointed to study this same problem. In this way the members

of the federation secure direct representation in the discussions of the N. E. A. on this matter.

The Committee on Geometry Syllabus has made great progress, and its report promises to be the best piece of work that has yet been done on this subject. The committee expects to finish its work during the coming year, and, when done, the report should be printed and distributed. Another committee is working on the chemistry unit, but this committee is not yet ready to report.

As a new topic worthy of consideration by the federation, the executive committee this year proposes the problem of methods of testing the results of teaching. In order to introduce this topic, Professor Judd was invited to give us some suggestions last year, at the Boston meeting, and Professors Thorndike and Caldwell will give us further ideas on this matter at this meeting. The executive committee believes that no greater benefit can be conferred on a teacher than to interest him in his teaching problems and to encourage him in the scientific study of them. With this idea in mind the Bibliography of Science Teaching was the first piece of work undertaken by the federation; and the papers presented at the meetings in Baltimore and Boston were planned to point in the same direction. Your committee now plans to follow with suggestions as to scientific methods of testing results, since on such methods depends the teacher's ability to find out definitely where improvement is possible.

Another line of investigation which the executive committee wishes to suggest to the federation is that of studying the relations of each particular science to the field of science teaching as a whole, including the problem of the general science course and that of four year science courses in high schools. It is evident that progress in the solution of these problems must precede progress in the solution of the problems of each particular science, and that the federation is in a far better position to carry on such study than is any one of the local associations by itself. We recommend this problem to the incoming executive committee as one to which the energies of the federation might be directed to advantage.

In a nut shell: The federation has been in existence for four years. It has cost the local associations not quite $400. It has

issued three bulletins, brought about better acquaintance and understanding among science and mathematics teachers the country over, been a means of producing several valuable reports, and paved the way for securing for the secondary school teachers a voice in college entrance questions and an interest in the scientific study of their teaching problems.

Yet, notwithstanding the value of the work thus far accomplished, your executive committee believes that the work of the federation is not yet fully understood by all the local associations. Experience has shown us that work of this kind must be done slowly, and the local associations are in many cases looking for quicker returns than is possible when so many interests are involved and everything has to be settled by correspondence. For example, nine months of constant correspondence among the members of this committee and the officers of local associations was required before this committee was able to settle the membership of the Committees on Relations between Colleges and High Schools and on a Syllabus in Geometry with the assurance that all the interests represented in the federation were fairly well cared for. In like manner, five months of correspondence was required to select the members of the new Committee on Logarithmic Tables.

For the same reasons, the work of the committees appointed has been slow and laborious. That results have been attained at all is due to the persistent and extensive work done by the chairmen and the members of these committees. Because of this work, however, science teachers are beginning to be better acquainted with one another, and the difficulty of the work will surely diminish as time goes on. The hardest part of the work has now been done, and the federation will be able with increasing efficiency to serve the cause of securing better conditions for teaching science, provided the local associations are ready to continue their cordial support of the organization. If such support fails, the work must grow more difficult and less effective.

During its term of service your committee has had very forcibly impressed upon it the fact that there are still many conflicting interests among the science teachers, and that work that must be carried on largely by correspondence must pro-

gress slowly. We are very sure that no other organization has yet produced reports more cosmopolitan and more suited to the needs of the times than those drawn up by the committees of the federation. The value of these reports cannot be appreciated fully until they have been at work for some time. Unless the local associations are still firm in their belief that the federation is being and will continue to be of great service to science teachers in enlarging and developing their common interests, we do not think that it is right to attempt to continue the organization and ask any five men to shoulder the work involved in membership in the executive committee. If the associations understand the work and encourage it, the work is well worth while.

Your executive committee therefore makes the following recommendations:

1. That this report and such other reports of committees as may be presented at this meeting be printed and distributed to all members of all associations as soon as possible.

2. That each association be requested to consider these reports very carefully, to consider the fact that the organization is now formed and the machinery established, for carrying on the work, and to decide, some time before the next meeting (December, 1911), whether it believes that work of the kind that the federation has shown that it can do, is, in the light of local needs, worthy of the continued loyal support of the association. In other words, that each association decide whether it wishes to continue to support the federation.

3. That in case an association decides that it is not wise to continue the Federation, that association be urged not to withdraw until all associations have reported on this matter and all have been notified how the vote stands and what the general feeling among the local associations is.

Report of the Committee on College Entrance Requirements in Mathematics and Science.

For your committee, appointed in June, 1909, to outline a policy on the part of the federation toward present entrance requirements in mathematics and science, the undersigned begs leave to submit the following report, with this word of explanation:

Inasmuch as an actual meeting of your committee has proven impossible, the outline of this report was submitted to the several members for criticism and suggestions, and advantage has been taken of these suggestions in making the final form accord as nearly as possible with the general sentiment of the committee. This renders the chairman alone responsible for the final form, and the report is therefore submitted for action, with this explanation.

Your committee has conducted much correspondence with secondary teachers in all parts of the country, and finds an ever increasing dissatisfaction with the present entrance conditions. This feeling has been well expressed in a pamphlet published by the High School Teachers' Association of the City of New York, entitled, "Articulation of High School and College; or, A Plea for the Reorganization of Secondary Education." In this pamphlet there is set forth in ample detail the rapidly spreading conviction that our public high schools are seriously hampered by present college entrance requirements. Copies of this pamphlet can be secured by applying to C. D. Kingsley, 400 Fourth Street, Brooklyn, N. Y.

In the West, school and college are coöperating with marked success in bringing about a better adjustment between them.

To aid in securing an equally happy condition in the East, the association appointed a committee of five, to whose careful study of present conditions is due the admirable presentation of facts, of practice, and of opinion contained in their pamphlet, which so well deserves full and thoughtful consideration from both school and college.

Further voice has recently been given to the same sentiment by the action of three important departments of the National Education Association. The departments of secondary education, of manual training and art, and of business education united at the last meeting in requesting the colleges to give to the matter of entrance requirements their most careful consideration.

In the judgment of your committee, the time is ripe for a formal submission of this whole subject to the various affiliated societies now allied to the federation, so that next year we may proceed to action with a full expression of opinion from them, and thus proper and necessary action may be taken.

Your committee has therefore drafted resolutions hereto attached, formulating their ideas of the needed changes in the matter of entrance requirements, and now recommends that these resolutions be referred to the various affiliated societies for their consideration and action during the coming year.

Respectfully submitted for the committee.

WILLIAM M. BUTLER,
Chairman.

YEATMAN HIGH SCHOOL,
ST. LOUIS, MO.

RESOLUTIONS SUBMITTED FOR ADOPTION.

WHEREAS—The recent extraordinary economic progress in the scientific and industrial worlds has created new and profound educational problems which can only be solved by careful investigation and experimentation, conducted by men experienced in the work of these schools, and with the aid and coöperations of the college, as well as of the business community, and

WHEREAS—The present high school courses have been subjected to trenchant criticism, especially from the industrial and business worlds, chiefly because present courses give insufficient attention to vocational training or to the future work of the child, and this has been one of the causes contributing to the loss from the high school of both boys and girls who would profit largely by courses that would more directly prepare them to meet the actual demands of business and of manufacturing life; and

WHEREAS—Although we recognize the grat benefits that have come in the past to the secondary school through college entrance requirements, we yet believe that the present excessive severity of these requirements along certain traditional lines and the failure of the colleges to recognize the educational value of vocational courses toward college admission, are conditions which very seriously hamper the freedom of the secondary schools and prevent necessary investigation, repeated experiment, and successful development of courses to meet present needs and educational growth; and

WHEREAS—The present "unit system" of admission to college, after sufficient trial, has proven unsatisfactory because of the difficulty of reaching fair and suitable definition of these

units and of properly crediting them, thus provoking endless discussion between school and college, as well as causing never ceasing friction in administration; and

WHEREAS—The colleges have long been urging the secondary schools to employ none but college graduates as teachers, and the schools have so far as the college offered trained teachers endeavored to comply with this request; therefore be it

Resolved—That we urge the colleges to abandon the "unit system" and in its place to accept the certificate of the high school at its face value for such work as it covers, and permit this to entitle the student to take up such college work as his preparation may warrant, whenever the certificate stands for four years of systematic and thorough training in a good high school; and

Resolved—That we recommend that the merits of the high school for such certification be determined by conference between schools and colleges in such associations as the North Central Association of Secondary Schools and Colleges, so that due weight may be given both to what the colleges desire and to what the schools can safely undertake; and

Resolved—That we request the colleges to consider whether the work done by its students in college does not in large part furnish a better basis for testing the efficiency of school preparation than do the present methods of entrance examination and of official inspection and

Resolved—That as we consider the larger and the more important duty of the secondary school is the preparation of the students for immediate entrance upon useful life in their own communities, we believe the college should cease to discriminate against subjects that the schools find necessary in preparing their pupils for such duties; and

Resolved—That we invite the college to come into more intimate contact with the secondary schools by requiring their professors who give general courses to do some visiting of the secondary schools at frequent intervals, so that they may acquire a better personal acquaintance with high school work and high school conditions and

Resolved—That we urge upon college men that they take a larger part in the work of teachers' associations, so as to secure

real acquaintance and earnest coöperation, based upon mutual consideration and esteem; and

Resolved—That we urge the colleges to offer greater facilities for the adequate training of teachers that liberal courses in pedagogy be established and ample opportunities be offered for practice teaching. Such influences for professional training, we believe, will do far more to raise the standard of work in our high schools than the present entrance requirements or system of inspection.

Report of the Committee on Logarithmic Tables.

Your committee to consider the question of the best form of logarithmic tables for use in college entrance examinations begs leave to submit the following report.

The committee began its work on the assumption that the principal question to be decided was whether on the whole four-place tables or five-place tables will best serve the purpose stated, and that tables of less than four places of decimals or more than five places are used so seldom in school work as not to require serious consideration in this connection. As a result of extensive correspondence with teachers both in colleges and secondary schools, the committee found that this assumption was fully justified. When it came to a choice between four-place and five-place tables, however, the committee found both sentiment and practice pretty evenly divided. Amongst the larger and more influential colleges and universities in the East and Middle West, both sides of the question are represented, with predominance, so far as numbers are concerned, in favor of five-place tables. It is to be noted, however, that with most of the colleges which admit students only by examination, the practice is to use four-place tables in the entrance examinations, and the sentiment in favor of this practice is with them very pronounced.

The secondary schools are much more strongly on the side of five-place tables. This is especially true of the public high schools, including the large city high schools, although there are exceptions. The private schools, whose business is predominatingly college preparation, are again pretty evenly divided. Some reported to the committee that they prefer and use five-place tables in their work, even though it involves the necessity

of giving many of their pupils special training in the handling of four-place tables for the college examinations. With many if not most of these schools, however, the desirability of uniformity of practice in the college examination outweighs other considerations.

On the whole the committee concludes that the sentiment in favor of the use of four-place tables in college entrance examinations is growing, though it has not yet by any means reached such a measure of predominance as would entitle it to exclusive recognition.

Three subordinate questions came up for the committee's consideration. These were:

(1) Whether or not the tables used in examination should be provided with tables of differences and proportional parts.

(2) Whether with four-place tables they should be arranged for the sexagesimal or decimal division of the degree.

(3) Whether negative characteristics when they occur, should be printed as such, or increased by 10.

With regard to (1) the committee is of the opinion that the tables provided at college entrance examinations should not be furnished with tables of differences and proportional parts. In reaching this decision the committee considers that many pupils use in school tables which are not provided with these helps, and that where this is not the case they will undoubtedly have had such training as would enable them to dispense with their use. Moreover, it is more confusing to a student unfamiliar with these auxiliary tables to have presented to him at an examination a table which is provided with them, than would be the case where the conditions are reversed.

With regard to (2) the committee has found that the greater number of those who favor the use of four-place tables prefer also to have them arranged for the decimal division of the degree.

Upon (3) the committee found that sentiment is generally in favor of the prevailing practice of printing the characteristics 9, 8, etc., in the trigonometric tables instead of the corresponding negative characteristics.

For the reasons set forth above the committee makes the following recommendations, viz.:

I. That in college entrance examinations involving logarithmic computation, both five-place and four-place tables be provided, and the student be given the option which he will use.

II. That with five-place tables the trigonometric functions be given for every minute of the quadrant, but that with four-place tables they be given for every tenth of a degree, except that for the first eight or ten degrees they should be given for every hundredth of a degree; that no tables of differences and proportional parts be provided; and that logarithms with negative characteristics have the characteristics increased by 10.

The committee begs that the federation will accept this report and discharge them from further consideration of the subject.

FLETCHER DURELL,
VIRGIL SNYDER,
JOHN H. DENBIGH,
WILLIAM B. CARPENTER,
EDWIN S. CRAWLEY, *Chairman.*

REPORT OF THE COMMITTEE ON BIBLIOGRAPHY.

Your Committee on Bibliography of Science Teaching begs to report that the Bibliography has been accepted for printing by the United States Bureau of Education. It is now in type, the proof has been read, and printed copies should be available at a very early date.

R. E. DODGE, *Chairman.*

REPORT OF THE COMMITTEE ON GEOMETRY SYLLABUS.

The Committee of Fifteen on Geometry Syllabus, which was appointed at the Baltimore meeting has been working in three subdivisions of five members each under the chairmanship respectively of Professor D. E. Smith, of Columbia University, Professor E. R. Hedrich, of the University of Missouri, and Professor H. L. Rietz, of the University of Illinois. The first division has under consideration definitions and logic, the second minimum and full list of theorems, and the third problems and applications. The work of each subcommittee has been fully developed and transmitted to all members of the whole committee for further criticisms and suggestions, previous to a

meeting of the subchairmen and several other members held at Cleveland on November 25 and 26, 1910, at which time the whole matter was worked over item by item and substantial agreement was reached on all points.

The plan is now to print a preliminary edition of two or three hundred copies to distribute among a selected list of teachers throughout the country for still further suggestions and criticisms previous to the presentation of a further report at the meeting of the National Educational Association in July, 1911, and the final report to the federation in December, 1911. It it believed that funds can be secured for printing the preliminary edition of the report, but it is hoped that the federation will be able to furnish the funds for its final distribution to practically all teachers of geometry in the country.

Respectfully submitted,
H. E. Slaught,
Chairman.

Report of the Committee on a Mathematical Journal.

The chairman of your committee on a mathematical journal has felt that the teachers of mathematics of this country should have a chance to say whether or not they desired a magazine devoted entirely to the interests of the teaching of mathematics. In order to get at this and other related questions, a questionnaire was sent to all members of our committee. Replies have been received from most members of the committee, but not, as yet, from all. For the most part those who did reply gave their opinions as to the feeling of the members of their associations without having the question presented and a census of the views of the members taken. This was due to the fact that the replies were sent before the various associations had held meetings. Your committee, therefore, has no complete report to make at this time. From the replies received, however, it does not seem to the chairman very likely that the committee will be able to come to any definite agreement, and if the federation thinks best it may perhaps be discharged.

W. H. Metzler,
Chairman.

Report of the Association of Teachers of Mathematics in the Middle States and Maryland.

This association has had a year of steady progress. The meetings of the general association and of its sections have been full of enthusiasm and of help for the teacher.

A number of committees have been at work on various problems, the one of broadest scope being, perhaps, that on the Algebra Syllabus. This committee has reported on Elementary, Intermediate, and Advanced Algebra, and the report has been amended and accepted by the association. This report will be published in the next number of the Mathematics Teacher.

A catalogue of members is in course of preparation. The names of those who are behind in their dues are being weeded out, and the membership is being put on as sound a basis as possible. The number of new members recently added is very large and will add considerably to the influence of the association.

The Mathematics Teacher has continued to gain in influence and in popularity, and there is no longer any doubt of its having proved the need of such a magazine.

The association has been invited to hold its annual meeting in conjunction with the Association of Colleges and Preparatory Schools of the Middle States and Maryland, and has accepted for next fall. The spring meeting will be held in New York.

Howard F. Hart,
Secretary.

Report of the Association of Mathematical Teachers in New England.

The association has held during the present year three formal meetings. The midwinter meeting at Hartford, Conn., February 26, was held in connection with the meeting of the Connecticut Association of Classical and High School Teachers. Here Mr. John C. Packard read an account of certain personal details in connection with John Perry, and gave an interesting survey of the main features of the Perry Movement as they appeared to him on his visit to England. Mr. Eugene Randolph Smith delivered by invitation an address on "The Syllabus

Method of Teaching Geometry," and Mr. George W. Evans spoke on "Simpson's Rule for Plane Areas."

The eighth spring meeting was held in Cambridge, Mass., April 16, and included two notable addresses—one on "Arithmetic for Industrial Schools," by Mr. William H. Dooley, of the Independent Industrial School of Lawrence, Mass.; and the other, "Mathematics for Agricultural Students," by Professor Charles A. Wheeler, of the Connecticut Agricultural College, Storrs, Conn. Six practical teachers gave their advice as to special methods of teaching algebra in the entering class of the high school.

The eighth annual meeting, held in Boston on Saturday, December 3, presented two addresses, one by Professor J. C. Tracy, of Yale University, on "Efficiency in Calculation," and the other by Mr. Peter F. Gartland, of the English High School in Boston, on "Two Experiments on Grammar School Graduates."

Most of the work of this year was devoted to examining possible adjustments between the modern demand for practical instruction and the scientific necessities of mathematical education. The addresses delivered before the association have borne great emphasis on practical applications and practical details of work. The association is not committed, by any means, to any of the different kinds of reform in mathematical instruction that have been brought forward, but a disposition to examine all such propositions has shown itself in many ways, notably in the appointment of a committee whose duty it is to consider the question whether there is any reason for remodeling the secondary school course in mathematics, in content or in order of development; this committee to report at some meeting of the association next year. The discussions that have taken place in the meetings have enlisted the attention of many of the college men who are members, and the association has had at various times the benefit of their judgment upon these matters. It is uncertain, however, whether the association will commit itself either one way or the other during the coming year.

GEO. W. EVANS,
Secretary.

Report of the Central Association of Science and Mathematics Teachers.

The Central Association of Science and Mathematics Teachers has performed a very profitable year's work. Committees appointed a year ago have carried on their investigations during the year, and made reports at the annual meeting.

Reports were presented to the association at the annual meeting in Cleveland, O., November 25 and 26, by a committee on "Fundamentals Common to the Various Sciences and Mathematics," by a committee on "Coöperative Experiments in Teaching Science," and by a committee on "The Relation of Elementary School Nature Study to Secondary School Science." An address was given by Dr. Dayton C. Miller, Case School of Applied Science, on "Sound Waves: Their Meaning, Registration, and Analysis." An address was given by Dr. Harvey W. Wiley, Washington, D. C., on "Food Facts which Every Citizen Should Know." Also many valuable reports and addresses were given before the various sections of the association.

All of these reports and addresses will be printed in the annual volume of Proceedings, which has been made a permanent publication of the Central Association.

The annual report of the secretary-treasurer for the year ending November 23, 1910, showed that the Central Association had at that date a paid-up membership of 488, and a total membership of 575, showing a large increase in the membership of the association during the year.

The next annual meeting will be held in Chicago in November, 1911.

Jas. F. Millis,
Secretary.

Report of the New England Association of Chemistry Teachers.

The thirty-sixth meeting of the association was held at Harvard University on December 4, 1909. After the reports of standing committees, the election of officers was held. Ostwald and Morse's "Elementary Modern Chemistry," Emery's "Elementary Chemistry," and Segerblom's "First Year Chem-

istry," were reviewed. Mr. L. G. Smith spoke on "Some Experiences of an American Teacher in the German Higher Schools." Professor T. W. Richards's address was on "The Value of Investigation to the Teacher of Chemistry."

The thirty-seventh meeting was held at the Lowell Textile School on February 12, 1910. The morning was spent in observing the equipment and the methods employed in the teaching of textile chemistry. At the afternoon session Professor Olney gave a very interesting talk on the work of the school. Godfrey's "Elementary Chemistry" and Biltz's "Introduction to Experimental Inorganic Chemistry" were reviewed. The report of the delegates to the American Federation meetings was made by Wilhelm Segerblom. Professor Newell presented the report of the committee on current events.

The thirty-eighth meeting was held at the Harvard Medical School on April 16, 1910. The meeting opened with the reports of the various standing committees. Under new business Mr. Segerblom called attention to the American Federation report in *School Science and Mathematics*. Attention was also called to the fact that delegates had been appointed by the federation to consider the possibility of changing the College Entrance Board chemistry syllabus. The principal address was on "The Function of Enzymes in the Chemistry of Life," by Dr. A. W. Peters. This was followed by a short address by Dr. C. A. Scott on "The Psychology of Science Teaching." The laboratories of the Medical School were visited at the close of the morning session. The afternoon was spent at the Carnegie Nutrition Laboratory, where Mr. Carpenter, the chemist in charge, described the laboratory and its special apparatus.

Through the kindness of President A. Lawrence Lowell, a course on the "Chemistry of Food" was given by Professor A. G. Woodman.

The twenty-ninth meeting of the association was held at Boston University, October 22, 1910. After the reading of the reports, Williams's "Essentials of Chemistry" and Meade's "Chemist's Pocket Manual" were reviewed. The opening address was given by Professor H. W. Morse on "The Experimental Basis of the Theory of Radio-activity." Mr. W. G. Whitman described some interesting new experiments. The

closing address was by Professor Latham Clark on "Some Phases of the German Chemical Industry."

The following officers were elected for the year 1910–11: President, Mr. F. C. Adams; vice-president, Mr. C. W. Goodrich; secretary, Mr. E. S. Bryant; treasurer, Mr. A. M. Butler; executive committee, Professor F. L. Bardwell, Mr. H. Bisbee, and Mr. G. A. Cowen.

EDWARD S. BRYANT,
Secretary.

REPORT OF THE MISSOURI SOCIETY OF TEACHERS OF MATHEMATICS AND SCIENCE.

Our society has held two meetings during the past year—one at Kirksville, Mo., in May, and one in connection with the State Teachers' Association at St. Joseph's, Mo., in November. We have made arrangements with the State Teachers' Association to have reports of our meetings published in connection with their annual report and to have reprints made for the use of members of the society and others who may wish them.

Our membership for 1909–10 was 114. Our total membership for this year cannot be determined at this time, since not all of our members have renewed their membership.

L. D. AMES,
Secretary.

TESTING THE RESULTS OF THE TEACHING OF SCIENCE.

BY EDWARD L. THORNDIKE,

Teachers College, Columbia University.

The topic which I am asked to discuss is one of enormous complexity. The changes in human beings which result from the teaching of science in schools are real, are measurable, and will some day be defined in units of amount as we now define changes in the rate of a moving body or in the density of a gas. But they include thousands of different elements; they vary with every individual; some of them can be demonstrated only long after school is completed; and at present units and scales in which to state changes in knowledge, power, interest, habits, and ideals are mostly matters of faith. An adequate measurement of the changes wrought in one class by one course in physics would be a task comparable to a geological survey of a state or an analysis of all the materials in this building.

I must also at once confess that I cannot bring you the results of specific investigation of educational achievement in science, but only such suggestions as general experience in measuring human faculty and various educational products can provide.

These suggestions fall naturally into two divisions according as one searches for means of measuring the specific information, skill, interest, and habits added by courses in science or the more general changes in total mental make-up, in, for instance, open-mindedness, accuracy, zest for verification, and the like.

The specific changes are, of course, the easier to measure. Indeed, my first suggestion is that we make scientific use of the measurements that we already make. For example, the regular school examinations are, or should be, careful scientific measures of important changes in our pupils. If we would test our classes with the examinations set by other teachers, have the pupils' work graded by other teachers, and print questions,

work, and grades, we should be making a start toward a real measurement of educational achievement.

If examinations are worth giving at all, they are worth giving, at least occasionally, in such a way as to measure not only how well a pupil has satisfied some particular person, but also what he really is or knows or can do in certain special fields.

We need thousands of significant questions, in each science, thousands of "originals" in physics, chemistry, and biology like the originals of geometry; and above all we need to have thousands of classes tested by outside examiners, as has been done in arithmetic, spelling, handwriting, and geography by Rice, Cornman, Stone, Earhart and Thorndike. If an examination, instead of being a hasty, subjective selection of questions graded still more personally (and alas, how hastily), were made a serious educational measurement, the examination papers of a year would alone give us a large start toward knowledge of what science teaching actually does.

Knowledge may, however, be measured more conveniently than by the examination of note-books, essays, or replies to questions of the ordinary sort. These have the merit of adequacy and richness, but the defects of measuring too many things at once and too indefinitely. Greater uniformity in the use of the test, quickness in scoring it, and freedom from ambiguity in the numerical value assigned can be secured by the exercise of enough ingenuity. I will mention two tests as samples of the many that are possible. The first is an adaptation of a test, devised by Ebbinghaus to measure mental efficiency in general, in filling in words omitted from a passage. From even a hastily devised sample presented here it will be seen that this form of test is scored with reasonable ease. The speed of an individual in selecting words to fill the gaps and the appropriateness of his selections together measure his knowledge. The former is scored with no effort at all and the latter with far less effort than is required to evaluate answers to questions, essays, or experimental works. The paragraphs and omissions therefrom should be arranged with care and improved after trial, but it may be of interest to some of you to compare the ratings obtained in six or eight tests of five minutes each like the following:

A body changing its position in space moves in a certain ....at a certain .... in the .... called acceleration. To change either the .... or the .... of a moving .... requires .... Suppose a pound of lead to be held at rest 500 feet above the surface of the ocean by a string and the string to be cut. The body will .... toward the .... of the .... beginning to .... with a .... of just barely over .... and reaching at the end of one second a .... of .... feet from where it started. In one second the .... will have .... from .... to .... feet per ....

The second is a very simple development of so-called association tests which I have used with good success in regular examinations in psychology for a number of years. It needs no other explanation other than a sample.

Write after each of these words some fact which it suggests to you.

acceleration,	gravity,	current,	lever,
density,	expansion,	elastic,	inclined.

As useful means of measuring the interests aroused by the study of science, I suggest records of the books taken from public libraries, of the periodicals chosen in public reading rooms, of the collections gathered and objects constructed by pupils, and a modified form of the test just described, the given words being much less easily provocative of thoughts about facts of science, and being mixed if necessary with words that would call up facts of science only in a person absorbed by scientific interests: The sample I give is left without such padding for disguise.

Write after each of these words some fact which it suggests to you.

work,	time,	wave,	square,	positive,
light,	level,	change,	water,	rate,
pull,	book,	mass,	study,	transform,
gas,	long,	contract,	heat,	law.

This latter test of interest should be varied using pictures of say a man rolling a barrel up a board into a wagon, a lightning flash in the sky, an ordinary (?, ???????) scales, and the like, with a similar mixture of "innocent" pictures. Besides words and pictures, actual or described events can be used. If such association tests are to be used to measure interest, they should not be used previously in the form calling definitely for facts about science.

These tests of interest may be used to measure both special interest in particular sciences and general interests, as in fact rather than fiction, knowledge rather than opinion, or verification rather than dispute. Of other means of measuring the general changes wrought by the study of science I will mention only two. The first concerns the power to utilize experience well in thought.

What is needed for this purpose is a series of problems or tasks, relative success with which depends as much as possible upon having power to use experience and as little as possible upon having had certain particular experiences. For example, relative success with the problem, "Which is heavier, a pint of cream or a pint of milk?" is determined largely by ability to select in thought the essential fact that cream rises and to infer its obvious consequence. The data themselves are possessed adequately by all or nearly all pupils alike.

To get such problems I wrote some time ago to one hundred teachers of science, half in universities and half in colleges. I quote some of them:

Rain drops are coming straight down. Will a car standing still or one moving rapidly receive in one minute the greater number of drops on its roof and sides?

Is air drawn up a hot chimney or is it pushed up?

Since it is possible for a person to float in water why is it possible for him to sink?

A cylinder and a cone equal in base and in altitude rest on a plane surface. Which is harder to tip over?

A magnet attracts two iron nails. If the magnet is removed will the nails attract each other?

It is harder to keep your hands clean in the winter than in the summer? Why?

How many surfaces, corners, and edges has a cube?

Which has the greater surface—a cube 10 inches on edge or a sphere 10 inches in diameter.

What is the largest mammal in the world?

Does an iron ball weigh more when it is hot than when it is cold?

If a bottle of gas which is lighter than air be placed with its open mouth upward, will the gas escape from the bottle or will the heavier air press the gas back into the bottle?

Is an incandescent lamp film on fire?

Will a ship that will just barely float in the ocean, float on Lake Erie?

Will a pound of popcorn gain or lose weight or stay the same after it has been popped?

The second means of measuring changes in general power to think is an adaptation of one devised by Professor R. S. Woodworth, in which the pupil picks out from such a series as that below, the statements that are logically absurd, not possibly true. It will be seen that statements could be chosen which would test the power of analysis and of thinking things together in any field of science from the most specialized to the most universal. Following is an example of this form of test.

Put a mark in the margin opposite each of the following sentences which is absurd:

Though armed only with his little dagger, he brought down his assailant with a single shot.

Silently the assembly listened to the orator addressing them.

While walking backwards he struck his forehead against a wall and was knocked insensible.

I saw his boat cleaving the water like a swan.

Having reached the goal, I looked back and saw my opponents still running in the distance.

Offended by his obstinate silence, she refused to listen to him further.

The one-armed cripple was attacked by a dog which seized his wrist, but he pushed it off with the other hand.

With his sword he pierced his adversary, who fell dead.

While threading my way through the crowd, I came suddenly upon an old friend.

The storm which began yesterday morning has continued without intermission for three days.

The dogs pursued the stag through flower gardens in full bloom.

That day we saw several ice-bergs which had been entirely melted by the warmth of the Gulf Stream.

While sharpening his three-bladed knife, my cousin cut his middle finger.

Our horse grew so tired that finally we were compelled to walk up all the hills.

The red-haired girl standing in the corner is taller than any of her older brothers.

A bricklayer fell from a new building quite near our house, and broke both his legs.

The hands of the clock were set back, so that the meeting was surely to close before sunset.

Many a sailor has returned from a long voyage to find his home deserted and his wife a widow.

The two towns were separated only by a narrow stream which was frozen over all winter.

The great advantages of these means of measuring intellectual ability lies in their rapidity and objectivity. If well devised, only two answers are possible, the pupil is measured easily, rapidly, and independently of subjective factors, and his condition is defined in terms of a simple numerical value.

There is no time for me to discuss methods of making, recording, and utilizing these or the hundreds of other equally worthy measurements of educational achievements, that is, of changes produced or prevented in human nature. Nor is this a proper occasion to outline the precautions that are required by the complexity and variability of facts of intellect and character and the absence of well-defined scales with equal units and known zero points, in which to measure facts of intellect and character. For our present purpose it is enough to know that, in spite of difficulties, the measurements can be made, and that a man of science can, if he will, be as scientific in thinking about human beings and their control by education, as in thinking about any fact of nature.

NEW BOOKS.

A Geometry for Schools. By F. W. SANDERSON and G. W. BREWSTER. Cambridge: The University Press; G. P. Putnam's Sons, American Agents. Pp. 346. $1.00 net.

The object of this book is to teach geometry by applying it to the solution of practical problems. Formal propositions and geometrical riders have a subordinate place. The examples form an important part of the book and seem to be well chosen. Algebra is freely used.

Cash and Credit. By D. A. BAKER. Cambridge: The University Press; G. P. Putnam's Sons, American Agents. Pp. 150. 40 cents net.

This is one of the volumes of the Cambridge Manuals of Science and Literature and is intended to give a good introduction to the subject of money and banking.

Solid Geometry. By H. E. SLAUGHT and N. J. LENNES. Boston: Allyn and Bacon. Pp. 194. 75 cents.

The two main purposes of this book, as stated by the authors, are:

1. That pupils may gain by gradual and natural processes the power and habit of deductive reasoning.

2. That pupils may learn to know the essential facts of elementary geometry as properties of space in which they live and not merely as statements in a book.

Elementary Arithmetic. By CHARLES W. MOREY. New York: Charles Scribner's Sons. Pp. 340.

This book is based upon the idea that number is essentially abstract, and that the prime object in the first years of school is to teach number as number. A minimum of theory and a maximum of practice are given to insure accuracy and facility.

The Psychology of the Emotions. By TH. RIBOT. New York: Charles Scribner's Sons. Pp. 474.

In Part I. the author treats of the general manifestations of feeling, pleasure and pain, and emotions. In Part II. he treats of special emotions and an attempt has been made to follow all the emotions one after another in the progress of their development, noting the successive movements of their evolution or their retrogression.

The False Equation. By MELVILLE M. BIGLOW. Boston: Little Brown and Co. Pp. 258. $1.50.

This is not a book on algebra, as the title might suggest, but a book on general education. The subject is considered from the side of the

state to carry out the trust charged upon it to establish and maintain equality as far as that is practicable in the government of men. The problem is one of providing men equal to the requirement, and that is essentially a matter of sound education. The author considers the difficulties to be overcome and the insufficiency of current modes of education to meet them, and then proposes a solution by what he terms organized education—the system by which the great and successful business corporations of the day are carried on.

Elements of Descriptive Geometry. By Albert E. Church and George M. Bartlett. New York: American Book Company. Pp. 286. Price $2.25.

This treatment of descriptive geometry, with applications to spherical projections, shades and shadows, perspective, and isometric projections, is for the use of technical schools and colleges. Though based upon Professor Church's "Descriptive Geometry," and retaining its original lucidity and conciseness, this work is fully up to date, and embodies present methods of teaching the subject. The figures and text are included in the same volume, each figure being placed beside the corresponding text, while many exercises for practice have been introduced. The old figures have been redrawn, and many of them have been improved. In the treatment of curved surfaces, all problems relating to single curved surfaces are taken up first, then those relating to warped surfaces, and finally those relating to surfaces of revolution.

First Year Algebra. By W. J. Milne. New York: American Book Company. Pp. 321. 85 cents.

Second Course in Algebra. By H. E. Hawke, William A. Lubby, and Frank C. Touton. Boston: Ginn and Company. Pp. 264. 75 cents.

NOTES AND NEWS.

The sixteenth meeting of the Association of Teachers of Mathematics in the Middle States and Maryland was held in Teachers College, New York, April 22, 1910. The meeting was called to order by the president, Dr. Wm. H. Metzler, at ten-thirty o'clock in the chapel of the college.

After the reading of the minutes, Mr. Breckenridge, chairman of the Committee on Continuation Schools, reported the progress of his committee. The report was very interesting in the matter of the attitude of the students in those schools for more pure mathematics merely because of their place in the curriculum of the ordinary day school. The report was accepted and the committee was continued. The Algebra Syllabus Committee was also continued.

The first paper of the morning was given by J. T. Rorer, of the William Penn High School, Philadelphia, on "The Curriculum: Present Tendencies, Future Possibilities."

The work of the morning was concluded by a paper by A. M. Curtis, of the Oneonta Normal School, on "Study Supervision: Its needs in the Mathematics of the Elementary and Secondary Schools."

The first paper of the afternoon was a description with lecture table models of the slide rule and its uses, by Clifford B. Upton, of Teachers College. This was followed by a description with stereopticon illustrations of the calculating machines then on exhibition in the Educational Museum of Teachers College.

Preliminary reports for the committees on arithmetic, algebra and geometry were given by Mr. Rorer for the Committee on Arithmetic and by Mr. Durrell for the Committee on Geometry. These reports consisted of plans for carrying on the work. The meeting adjourned (after expressing its thanks to Teachers College) to the Educational Museum for the privilege of inspecting the exhibit of slide rules, calculating machines, rare books and manuscripts.

The tenth regular meeting of the Rochester Section was held at the University of Rochester, Saturday, April 29, 1911, with the following program: "The Effort to Obtain a National

Syllabus for Geometry," by Mr. W. Betz, East High School, Rochester; "The Course in Mathematics at the Buffalo Central High School," by Miss M. M. Wardwell, Central High School, Buffalo; "A High School Mathematical Club," by Miss E. M. Pierce, High School, Lockport; "The Infinite in Geometry," by Mr. A. S. Gale, the University of Rochester.

THE regular spring meeting of the New York section was held Friday, May 12, 1911, at 8 P.M., in High School of Commerce with the following program: "The Way to Begin Solid Geometry," by Howard F. Hart, High School, Montclair, N. J.; "Testing Methods of Factoring the Quadratic Trinomial," by Fiske Allen, Horace Mann High School, New York City; "How to Decrease the Mortality in the First Two Years of the High School," by William R. Lasher, Erasmus Hall High School, Brooklyn; "A Boys' Mathematical Society," by William E. Breckenridge, Stuyvesant High School, New York City; "The Teaching of Algebra in the Grammar Grades," by James A. Bridges, High School, New Rochelle, N. Y.; "An Experiment in Parallel Courses," by Clarence P. Scoboria, Polytechnic Preparatory School, Brooklyn; "Alligation—Past, Present, and Future," by Willard A. Ballou, Pratt Institute, Brooklyn.

THE spring meeting of the Philadelphia section was held in the Central High School, Thursday afternoon, May 25. Two papers were presented: I., "A New Solution of Higher Equations, giving all roots, real and imaginary, expressed in Infinite Series," by Mr. H. H. Foering, headmaster of the Bethlehem Preparatory School. The discussion was opened by Professor Walter R. Marriott, Swarthmore College. II., "To What Extent should Graphical Methods be used in the Secondary School Algebra?", by Professor O. E. Glenn, University of Pennsylvania. This was discussed by Mr. Thomas Moore, North East Manual Training High School; Miss Muriel Smith, William Penn High School; Mr. Thomas K. Brown, Westtown School. The following officers were elected for the ensuing year: President, Dr. Edward D. Fitch, Delancey School; vice-president, Professor Maurice J. Babb, University of Pennsylvania; secretary, Miss Elizabeth B. Albrecht, Philadelphia High School for Girls; members of the executive committee, Miss Agnes Long, William Penn High School, Professor Charles L. Thornburg, Lehigh University.